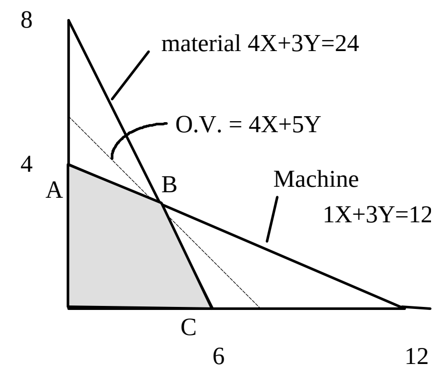
Production & Operations Management

Graphical Linear Programming Lecture Example Dr. Banis



Objective Function: MAX 4X + 5Y

Subject to:

Constraints: Machine time: 1X + 3Y <= 12

Material: 4X + 3Y <= 24

increase the objective value by moving the objective function line out from the origin. The farthest it can go while still having a point in the feasible solution space is the intersection of the constraints, Point B. If the slope were different, the last point of intersection could be A or C instead of B. The boundaries between the solutions are the conditions where the Objective Function is parallel to (has the same slope as) each of the constraints.

Algebraic Solution: *Three corners to consider:*

Point A: X=0; Y=4; OV = 4(0) + 5(4) = $20
Point C: X=6; Y=0; OV = 4(6) + 5(0) = $24
Point B (Intersection of the constraints):

1X+3Y=12	<u>Substituting:</u>	
<u>-1 * (4X+3Y=24)</u>	1X + 3Y = 12	O.V. = 4X + 5Y
-3X = -12	1(4)+ 3Y = 12	= 4(4) +5(8/3)
X = 4	Y = 8/3	O.V. = $29.33 ***

<u>MAX profit:</u>
B is the Optimal solution:
X =4; Y= 8/3
O.V. = $29.33

<u>Slacks:</u>

<u>Machine:</u>	1X + 3Y <= 12	<u>Material:</u>	4X + 3Y <= 24
	1(4)+3(8/3)=12		4(4)+3(8/3)=24
	No slack, all used		No slack, all used

Ranges of Optimality on Objective Function (cost/profit) Coefficients, 4X + 5Y:

If profitability of one of the products changes enough, the solution shifts to another corner. The changeover is when the Objective Function line is parallel to a limiting constraint. When the ratio of coefficients is the same in the O.F. as it is in the constraint, the optimal O.F. is a line that lies right on the constraint line, and there are an infinite number of equally good point solutions on this line, including the two corners that are bounded by this constraint. Because there isn't one unique solution, this solution is called *degenerate.*

<u>Effect of changing the Coefficient of Y on the Objective (Profit) function Value</u>

Corner	Product mix	4X + 2Y	4X + 3Y	**4X + 5Y**	4X + 12Y	4X + 13Y
A	Y = 4 ; X = 0	8	12	20	48	52
B	Y=8/3; X = 4	21.33	24	29.33	48	50.67
C	Y= 0; X= 6	24	24	24	24	24

Limits of Optimality for Coefficient X (Keeping coefficient of Y constant at 5):

<u>Parallel to Material Constraint when:</u>

$$\frac{CoeffX,O.F}{Coeff.Y,O.F.} = \frac{CoeffX,mat}{CoeffY,mat}$$

$$Coeff.X, O.F = \frac{CoeffX,mat}{CoeffY,mat} * Coeff.Y, O.F$$

$$Coeff.X, O.F = \frac{4}{3} * 5 = 6.67$$

<u>Parallel to Machine Constraint when:</u>

$$\frac{CoeffX,O.F}{Coeff.Y,O.F.} = \frac{CoeffX,mach}{CoeffY,mach}$$

$$Coeff.X, O.F = \frac{CoeffX,mach}{CoeffY,mach} * Coeff.Y, O.F$$

$$Coeff.X, O.F = \frac{1}{3} * 5 = 1.67$$

Range of Optimality for corner B, coefficient of X:
Upper Limit = $6.67
Lower limit = $1.67

Limits of Optimality for Coefficient Y (Keeping coefficient of X constant at 4):

<u>Parallel to Material Constraint when:</u>

$$\frac{CoeffY,O.F}{Coeff.X,O.F.} = \frac{CoeffY,mat}{CoeffX,mat}$$

$$Coeff.Y, O.F = \frac{CoeffY,mat}{CoeffX,mat} * Coeff.X, O.F$$

$$Coeff.Y, O.F = \frac{3}{4} * 4 = 3$$

<u>Parallel to Machine Constraint when:</u>

$$\frac{CoeffY,O.F}{Coeff.X,O.F.} = \frac{CoeffY,mach}{CoeffX,mach}$$

$$Coeff.Y, O.F = \frac{CoeffY,mach}{CoeffX,mach} * Coeff.X, O.F$$

$$Coeff.Y, O.F = \frac{3}{1} * 4 = 12$$

Range of Optimality for corner B, coefficient of Y:
Upper Limit = $12
Lower limit = $3

Outside these limits of profit per unit, the optimal production plan changes to another corner. Within these ranges of optimality, the total profit may change, but the optimal production plan stays the same, as the corner B solution.

Shadow price: Value of making one more unit of a constraining element available. Effect on profit or reduced cost caused by relaxing a constraint.

New intersection corner B with one more unit of material:

$1X+3Y=12$	Substituting:	
$-1 * (4X+3Y=25)$	$1X + 3Y = 12$	O.V. = $4X + 5Y$
$-3X = -13$	$1(4.33)+ 3Y = 12$	$= 4(4.33) + 5(2.56)$
$X = 4.33$	$Y = 2.56$	O.V. = \$30.11

Shadow price for material:
new O.V.=\$30.11
old O.V.=\$29.33
increase = \$0.78

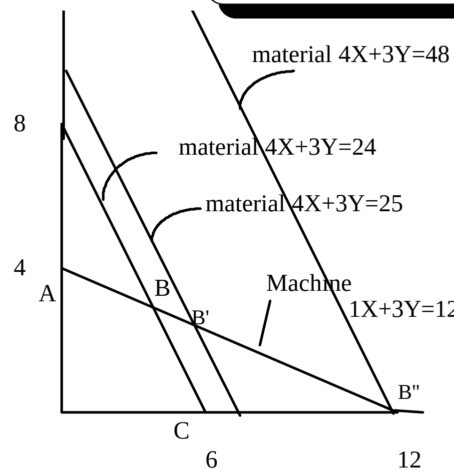
New intersection corner B with one more unit of Machine time:

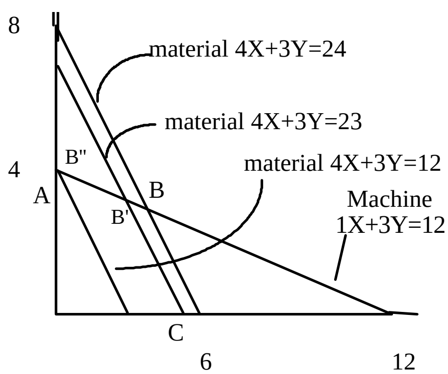
$1X+3Y=13$	Substituting:	
$-1 * (4X+3Y=24)$	$1X + 3Y = 13$	New O.V. = $4X + 5Y$
$-3X = -11$	$1(3.67)+ 3Y = 13$	$= 4(3.67) + 5(3.11)$
$X = 3.67$	$Y = 3.11$	New O.V. = \$30..22

Shadow price for machine time:
new O.V.=\$30.22
old O.V.=\$29.33
increase = \$0.89

Ranges of Validity:

Shadow prices are valid "until something changes." such as the end of an intersecting constraint. In the diagram, **increasing the amount of material** available from 24 to 25 units shifts the solution at the intersection from corner B to corner B'. the increase in value for this new solution is the shadow price. This shadow price is valid as long as the profit is changing at the same rate for every new unit of material, that is, as long as the same two constraint lines intersect. when the intersecting machine constraint line runs out at B", the next increment of material will have a different effect. At this point, X=12, Y=0, and material use is 4X +3Y =4(12) +3(0) = 48 units. having more material than 48 units would do us no good, as there wouldn't be enough machine time to let us use more than 48 units of material. Although each unit of material up to that point increases our profit by \$0.78, The 49th unit has a value of \$0, because we can't do anything with it. Thus, the shadow price of \$0.78 is only valid up to 48 units (24 units more than we started with).





Reducing the amount of material available from 24 units to 23 units would reduce our profit by \$0.78. this reduction per unit given up would continue until the the material constraint intersects corner A. At point A, we'd be making 4 units of Y, using 3units of material for each one for a total of 3*4=12 units of material. At that point, machine time is no longer constraining, and so each unit change in material will have a larger effect. Since we'd be making all Y, and each unit of material is enough to make 1/3 unit of Y, the reduction in profit would be 1/3 of the profit on a unit of Y, or \$5*(1/3)= \$1.67 per unit of material. Within the range where shadow price is \$\$0.78, we would be willing to sell some of our material as long as we were paid as at least as much as the profit we would lose by not having it available. We would sell material for any price greater than or equal to the shadow price of \$0.78 per unit. **Thus, the shadow price on material of \$0.78 is valid to an upper limit of 48 and to a lower limit of 12.**

Similarly, for the shadow price on machine time:

The ends of the material constraint define the limits on the the range of validity.
Upper limit: Y=8, X=0; material = 1X+3Y=1(0) + 3(8) = 24
Lower Limit: Y=0, X=6; material = 1X + 3Y = 1(6) + 3(0) = 6
Therefore, the range of validity on the \$0.89 shadow price on machine time is 6 units to 24 units.
If the optimal solution were at corner A rather than corner B, then the shadow price would be determined simply by the amount of Y that could be made by having the additional time (Note, material wouldn't be limiting at corner A unless we only had 12 units). No X would be made at corner A . The upper limits on the ranges of validity would be defined by the points at which the other resource runs out. The lower limits would be zero (where there is no more to sell).