Analysis of Variance

Comparing two means:

- H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$
- Compare difference between sample means vs. a pooled estimate of standard deviation.
- Use Z test or t test depending on whether σ is known or sample size > 25.

Comparing more than two means:

- Repetitive t or Z tests between pairs of means
- Cumbersome and gives "overtesting".
- At the 5% level, 1/20 comparisons would be "significant" just by chance. Must adjust for this.

ANOVA

Compares several means at once

- H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: one or more is/are different
- Compare reduction in estimate of variance for separating into more than one distribution with different means. Compare vs. a pooled estimate of variance remaining.
- Use F test with degrees of freedom:
- d.f. total =n - 1
- d.f. between = number of groups (k) - 1
- d.f. within = remaining d.f. = n - k
- compare to F tables for d.f.'s, significance levels.

Assumptions:

- Observations are independent
- variance is homogeneous across groups.

Procedure:

- Calculate total sum of squares, SSt= Sum(Xi^2)-(Sum $X)^2$/N
- Calculate between-group sum of squares, SSb
- Derive within-group SSw by SSt- SSb
- Divide by appropriate d.f. to get mean squares, MSw, MSb
- F = MSb/MSw
- Compare to critical F values table for dfb,dfw, significance
- Reject H_0 if F is big.
- Compare differences between individual pairs of means using Tukey's HSD test as a cutoff
- Tukey's HSD is "Honestly Significant Difference"--an adjustment of the "critical t" to allow for sloppiness when doing multiple comparisons.

$$HSD = q(\alpha, k, N-k)\sqrt{\frac{MSw}{n}}$$

- q from table in appendix C
- α is significance level, 1% or %5
- k is number of groups/treatments
- N is TOTAL observations
- n is number in each group
- MSw is within (error) mean square

- The Tukey test will tell you whether your conclusion about the differences will fly.

Mechanics--just like the computational formula for variance:

- Separate observations into groups by treatment.
- Calculate totals by group and overall (grand sum)
- Calculate means by treatment. Square each mean, multiply the square * the number of observations in each mean and add all these together.
- Square the grand sum and divide by the grand number = "Correction term"
- Square every observation and sum the squares (grand sum of squares)

Calculate corrected sums of squares:

- Total sum of squares, SSt = grandsumofsquares - CT
- Treatment Sum of Squares,
  SSb = Sumofmeans (*n in each) squared - CT
- Within (Residual) SSw = difference, SSt - SSb.
- Divide by appropriate df's to get Mean Squares
- Test ratio of Treatment mean square to within mean square.
- compare to critical F (α, dfb, dfw) from appendix B tables.

Multidimensional comparisons:

- Symmetrical Square designs.
- Same thing, except you regroup the observations by another treatment as well, and go through the separation out of another treatment sum of squares.
- These other dimensions "explain" more of the error variance and reduce the residual "unexplained" variation that's attributed to Within group random error.
- Degrees of freedom are also partitioned into the new explained dimension (df=number of groups-1) reducing the degrees of freedom associated with "error" variance.
- Since this further partition of variance reduces the variation attributed to random effects, it makes it easier to show differences that are attributable to causes.
- "Within group" = "Error" = "Random" = "Residual" = "Unexplained".
- Random error" isn't without cause. It only means we can't explain (don't know) the cause.